Different class of two-dimensional shocks in magnetized plasmas

A. Fruchtman[1] and L. I. Rudakov[2]
[1]Department of Physics, Weizmann Institute of Science, Rehovot 76100, Israel
[2]Kurchatov Institute, Russian Research Center, 123182 Moscow, Russia
(Received 21 April 1994)

A different class of two-dimensional shocks in magnetized plasmas, with a characteristic velocity that is larger than the hydrodynamic velocity, is described. At the one-dimensional limit shocks in cold magnetized plasmas are recovered. At the opposite limit the recently discovered [Phys. Rev. Lett. **69**, 2070 (1992)] fast two-dimensional shocks are recovered. The fast penetration of the magnetic field is induced by a density gradient along the current lines that is formed by the magnetic field pressure in the two-dimensional flow. If the collisionality is low, the electron heating might be small. The dissipated magnetic field energy is then converted to electron kinetic energy not in the form of thermal energy but rather as a directed energy that is convected away.

PACS number(s): 52.35.Mw, 52.35.Tc, 52.75.Kq

I. INTRODUCTION

In one-dimensional (1D) shocks in plasmas, as a result of the frozen-in law, a small magnetic field in the shock upstream (the unperturbed plasma) and a large magnetic field in the shock downstream are accompanied by a large plasma compression [1]. An unmagnetized upstream plasma and a magnetized downstream plasma imply an infinite plasma compression. The velocity of the 1D shocks is on the order of the hydrodynamic velocity.

It has been recently shown that high velocity shocks, in which the upstream magnetic field is small (and even zero) and the downstream magnetic field is large while the plasma compression is small, may propagate in plasmas that are nonuniform [2–9]. The nonuniformity may be a density gradient or magnetic field curvature. Both gradient and curvature lie in a direction perpendicular to the direction of shock propagation, reflecting the two-dimensional (2D) nature of the propagation. The velocity of those shocks was shown to be inversely proportional to the characteristic length of nonuniformity and to be substantially larger than the velocity of the 1D shocks, when this length is much smaller than the ion skin depth.

The high velocity shocks described above propagate only in plasmas that are nonuniform from the start. In a recent paper [10] we have shown that 2D shocks of a large magnetic field downstream may propagate into an unmagnetized plasma even if the plasma is initially homogeneous. The velocity of such 2D shocks was also shown to be substantially larger than the velocity of the 1D shocks. The plasma compression in the 2D shocks is small. The fast penetration of the magnetic field is induced by a density gradient along the current lines that is formed by the magnetic field pressure in the 2D flow. In this paper we present a unified analysis of the 1D and 2D shocks that propagate in plasmas that are initially homogeneous. The shocks we analyze are reduced at one limit to the 1D shocks, and at the other limit to the recently discovered [10] fast 2D shocks.

For simplicity we restrict ourselves to cases in which the plasma pressure is much smaller than the magnetic field pressure. In the case of 1D shocks this may be mainly when the shocks are weak (when the relative change of the magnetic field across the shock is small) if the upstream plasma is cold. We show, however, that in the case of 2D shocks the plasma pressure may remain small even if the shock is strong (the ratio of the downstream magnetic field to the upstream magnetic field is large). Therefore, the simplifying assumption of a small plasma pressure actually holds in this important case, as will be shown *a posteriori*.

We are interested especially in the low-collisionality case. As expected, when the collisionality is small, the shock structure is determined mainly by the electron inertia. The shock transition includes damping nonlinear waves. At the 1D limit of the general 2D shocks, the weak shocks in magnetized plasmas studied by Sagdeev [11] are recovered. In addition to their higher velocity, some of the 2D shocks are characterized by a low rate of electron heating. If the collisionality is small, the dissipated magnetic field energy is converted to electron kinetic energy not as a thermal energy, but rather as a directed kinetic energy that is convected away.

The propagation of high velocity shocks in plasmas that are nonuniform from the start has been suggested as the mechanism of magnetic field penetration into the plasma in the plasma opening switch (POS) [12,13]. It has been shown that such shocks have to be accompanied by a large electron heating [8]. Experiments indicate that such a large heating does not occur [14]. The fast penetration of the magnetic field observed in the POS, that is not accompanied by a large electron heating, could therefore be similar to the magnetic field penetration in the form of 2D shocks described here.

In Sec. II we present the model equations. In Sec. III we discuss in general terms the evolution of the energy in the system. In Sec. IV we find the shock velocity and the shock structure. The fraction of energy that becomes electron thermal energy is calculated in Sec. V. Cases are presented where this fraction is very small. Numerical examples and conclusions are given in Sec. VI.

1063-651X/94/50(4)/2997(9)/$06.00

II. THE MODEL

We examine the evolution in a plasma of a magnetic field, the only nonzero component of which lies in the direction of the ignorable coordinate y. Therefore, the magnetic field is of the form $\mathbf{b}=\hat{\mathbf{e}}_y b(x,z,\tau)$. The governing equations are Faraday's law

$$\frac{\partial b}{\partial \tau}=\frac{\partial E_z}{\partial x}-\frac{\partial E_x}{\partial z} , \tag{1}$$

Ampere's law

$$n v_x=\frac{\partial b}{\partial z} , \tag{2a}$$

$$n(V_z-v_z)=\frac{\partial b}{\partial x} , \tag{2b}$$

the ion continuity equation

$$\frac{\partial n}{\partial \tau}+\frac{\partial}{\partial z}(nV_z)=0 , \tag{3}$$

the momentum equation

$$n\frac{dV_z}{d\tau}=-b\frac{\partial b}{\partial z} , \tag{4}$$

and the electron momentum equation

$$\epsilon\frac{dv_x}{d\tau}=-E_x+v_z b-\nu(v_x-V_x) , \tag{5a}$$

$$0=-E_z-v_x b . \tag{5b}$$

The equations are in a dimensionless form, where b is the magnetic field normalized to B_0, n is the density normalized to the upstream density n_0, τ is the time normalized to the ion cyclotron period $\omega_{ci}^{-1}(\equiv Mc/eB_0$, where M and e are the ion mass and charge and c the velocity of light in vacuum), the coordinates x and z are normalized to the ion skin depth $c/\omega_{pi}[\equiv(Mc^2/4\pi n_0 e^2)^{1/2}]$, $\mathbf{v}$ and $\mathbf{V}$ are the electron and ion velocities normalized to $V_A(\equiv c\omega_{ci}/\omega_{pi})$, $\mathbf{E}$ is the electric field normalized to $B_0 V_A/c$, ϵ is the electron mass m normalized to the ion mass, and ν is the collision frequency normalized to eB_0/mc. In writing the equations we assumed that the derivatives with respect to z are larger than the derivatives with respect to x, so that L_z/L_x, v_z/v_x, and V_x/V_z are all small. By L_x and L_z we denote the characteristic lengths in the two directions. In Eq. (4), $d/d\tau\equiv\partial/\partial\tau+V_z(\partial/\partial z)$, while in Eq. (5) $(d/d\tau)\equiv(\partial/\partial\tau)+\mathbf{v}\cdot\nabla$. Quasineutrality is assumed and the displacement current is neglected.

The various velocities therefore satisfy the relations

$$V_x \ll V_z \le 1 \le u \le v_x . \tag{6}$$

Here u is the velocity of penetration of the magnetic field. The electron velocity v_z is much smaller than v_x.

We assume that the plasma pressure is negligible. This assumption restricts the domain of validity of our equations. The assumption is correct in the case of weak 1D shocks treated by Sagdeev [11]. It will also be shown to be valid in important 2D shocks with low heating.

III. THE PARTITIONING OF POWER

In this section we examine the partitioning of power in the plasma. We use the equations of Sec. II to derive equations for the evolution of the magnetic field energy, the electron kinetic energy, and the ion kinetic energy.

Multiplying Eq. (1) by b and using Eqs. (2a), (5b), and (6), we obtain an equation for the evolution of the magnetic field energy:

$$\frac{\partial}{\partial \tau}\left[\frac{b^2}{2}\right]+\nabla\cdot\mathbf{E}\times\mathbf{b}=V_z b\frac{\partial b}{\partial z}+\mathbf{E}\cdot n\mathbf{v} . \tag{7}$$

The first term on the left-hand side (LHS) represents the change in time of the magnetic field energy density, and the second term represents the divergence of the Poynting flux. The first term on the right-hand side (RHS) represents the work done by the fields on the ions, while the second term represents the work done on the electrons.

We now restrict ourselves to solutions of the equations in the form of a traveling wave. We therefore assume that

$$\frac{\partial}{\partial \tau}=-u(x)\frac{\partial}{\partial z} . \tag{8}$$

The waves propagate in the z direction with a velocity $u(x)$ that depends on x. From Eqs. (3) and (4), it follows that

$$n=\frac{2u^2}{[2u^2-(b^2-b_u^2)]} , \tag{9}$$

and that

$$V_z=\frac{(b^2-b_u^2)}{2u} . \tag{10}$$

Here b_u is the upstream magnetic field at the limit $z-u(x)\tau\to\infty$. In this general analysis we allow an arbitrary plasma compression and do not restrict ourselves to $n\cong 1$, as we previously did [10].

Using Eqs. (8), (9), and (10), we integrate Eq. (7) with respect to z, and obtain

$$u\left[\frac{b^2-b_u^2}{2}\right]-E_x b-\frac{\partial}{\partial x}\int_z^\infty dz' E_z b$$
$$=-\frac{1}{8u}(b^2-b_u^2)^4-\int_z^\infty dz'\mathbf{E}\cdot n\mathbf{v} . \tag{11}$$

The first term on the LHS represents the change in time of the magnetic field energy, the second term is the axial flux of the magnetic field energy, and the third term is the integral with respect to z of the radial flux of the magnetic field energy. The first term on the RHS is the rate of work done on the ions, and the second term is the rate of work done on the electrons.

Let us examine the term associated with the radial flux of the magnetic field energy, the third term on the LHS of Eq. (11). Using Eqs. (2a), (5b), and (9), we perform the integration with respect to z:

$$\frac{\partial}{\partial x}\int_z^\infty \frac{dz'}{n}\frac{\partial b}{\partial z'}b^2$$

$$=-\frac{\partial}{\partial x}\left[\frac{(b^3-b_u^3)}{3}\left[1+\frac{b_u^2}{2u^2}\right]-\frac{(b^5-b_u^5)}{10u^2}\right] . \quad (12)$$

This term vanishes if the flow is 1D, i.e., if there is no dependence on x. If the flow is 2D, i.e., if the dependence on x is not zero, this term contributes substantially. The fast evolution of the magnetic field is due to this very dependence of the radial flux of the magnetic field energy on x.

We now assume a shock propagation in which for $z-u(x)\tau\rightarrow-\infty$, at the shock downstream, $b\rightarrow-1$, $v_x\rightarrow 0$, and $v_z\rightarrow V_z$. Using Eqs. (5a) and (10) for expressing E_x and Eq. (12), we write Eq. (11) as

$$P_z+P_x=U_B+U_i+U_e , \quad (13)$$

where

$$P_z=\frac{(1-b_u^2)}{2u} \quad (14a)$$

is the net axial flux of magnetic field energy,

$$P_x=-\frac{d}{dx}\left[\frac{1}{u^2}\right]\left[b_u^2\frac{(1+b_u^3)}{6}-\frac{(1+b_u^5)}{10}\right] \quad (14b)$$

is the radial flux of magnetic field energy integrated with respect to z,

$$U_B=u\left[\frac{1-b_u^2}{2}\right] \quad (14c)$$

is the rate of increase of magnetic field energy,

$$U_i=\frac{1}{8u}(1-b_u^2)^4 \quad (14d)$$

is the rate of work done on the ions, and

$$U_e=\int_{-\infty}^{\infty} dz\,\mathbf{E}\cdot n\mathbf{v} \quad (14e)$$

is the rate of work done on the electrons. Note that once $u=u(x)$ is specified, the partitioning of the power (between the increase of the magnetic field energy, the work on the ions, and the work on the electrons) is determined.

We now follow the partitioning of the magnetic field energy dissipated as a work on the electrons. Using Eqs. (2), (3), and (5), we derive an equation for the evolution of the electron kinetic energy:

$$\frac{\partial}{\partial\tau}\left[\epsilon\frac{nv^2}{2}\right]+\nabla\cdot\left[\epsilon\frac{nv^2}{2}\mathbf{v}\right]=-\mathbf{E}\cdot n\mathbf{v}-\nu n\mathbf{v}\cdot(\mathbf{v}-\mathbf{V}) . \quad (15)$$

We assume that the relation described by Eq. (8) holds, integrate Eq. (15) with respect to z across the shock, and obtain

$$U_e=A+uQ , \quad (16)$$

where

$$A\equiv\int_{-\infty}^{\infty} dz\frac{\partial}{\partial x}\left[\frac{\epsilon}{2n^2}\left[\frac{\partial b}{\partial z}\right]^3\right] \quad (17a)$$

and

$$Q\equiv\int_{-\infty}^{\infty} dz\frac{\nu}{un}\left[\frac{\partial b}{\partial z}\right]^2 \quad (17b)$$

is the rate of Joule heating. As we mentioned above, when b_u and u are specified, U_e is determined. However, the partitioning of U_e between energy that is convected away (the first term in the integrand) and energy converted to thermal energy through Joule heating (the second term in the integrand) is not yet determined.

In order to pursue the analysis of the energy flow we need to find an expression for $d/dx(1/u^2)$ in Eq. (14b). To do that, we regress temporarily to the equations of Sec. II. Combining Eqs. (1)–(6) and (8)–(10), we obtain

$$\frac{\epsilon}{n}\frac{\partial}{\partial z}\left[\frac{1}{n}\frac{\partial b}{\partial z}\right]-\frac{\epsilon}{u}\left\{b,\frac{1}{n}\frac{\partial b}{\partial z}\right\}-\frac{\nu}{un}\frac{\partial b}{\partial z}+F(b)=0 , \quad (18)$$

where

$$\{f_1,f_2\}\equiv\frac{1}{n}\frac{\partial f_1}{\partial z}\frac{\partial f_2}{\partial x}-\frac{1}{n}\frac{\partial f_2}{\partial z}\frac{\partial f_1}{\partial x} \quad (19)$$

are Poisson's brackets, and

$$F(b)\equiv-(b-b_u)+\frac{(b^2-b_u^2)}{2u^2}b-\frac{d}{dx}\left[\frac{1}{u^2}\right]\frac{(b^2-b_u^2)^2}{8u} . \quad (20)$$

The second term in Eq. (18) results from the convective derivative in the electron momentum equation, and it has this form because of the derivation with respect to x. The dependence on x is also important in the expression for F through the presence of the third term on the RHS of Eq. (20). Since b is uniform in the shock downstream, it follows from Eq. (18) that

$$F(b=-1)=0 , \quad (21)$$

or, explicitly,

$$\frac{(-1+b_u)^2(1+b_u)}{8u}\frac{d}{dx}\left[\frac{1}{u^2}\right]=1-\frac{(1-b_u)}{2u^2} . \quad (22)$$

Equation (22) provides us with the expression we need for $d/dx\,(1/u^2)$, and we may continue the analysis of the energy flow. Substituting the expression for $d/dx\,(1/u^2)$ from Eq. (22) into Eq. (14b), we find that the radial flux of magnetic field energy, integrated with respect to z, is

$$P_x=\left[\frac{b_u^2(1+b_u^3)}{6}-\frac{(1+b_u^5)}{10}\right]\left[\frac{(1-b_u)}{2u^2}-1\right]\times\frac{8u}{(1-b_u)^2(1+b_u)} . \quad (23)$$

A. The 1D limit

We assume that there is no dependence on x. These are the usual 1D shocks with the assumption of a zero plasma pressure. These shocks, in the case of low collisionality, have been studied in detail by Sagdeev [11]. Using Eq. (22), we find that the shock velocity is

$$u = u_s \equiv \left[\frac{1-b_u}{2} \right]^{1/2} . \tag{24}$$

Since there is no dependence on x, the fluxes are

$$P_x = 0 \tag{25a}$$

and

$$P_z = (1+b_u)u_s . \tag{25b}$$

We examine the two cases, a weak shock in which the upstream magnetic field and downstream magnetic field are similar, and a strong shock, in which the upstream magnetic field is zero. We start with the weak shock:

$$1+b_u \ll 1 . \tag{26}$$

The flux of magnetic field energy is divided as follows:

$$\frac{U_B}{(P_x+P_z)} \cong 1 \tag{27a}$$

and

$$U_i, U_e \ll U_B . \tag{27b}$$

The magnetic field energy goes mainly to build the magnetic field energy in the shock downstream, and only a small part is dissipated as work on the ions or on the electrons. If the plasma upstream is unmagnetized,

$$b_u = 0 , \tag{28}$$

the power is divided as follows:

$$\frac{U_B}{(P_x+P_z)} = \frac{1}{2} , \tag{29a}$$

$$\frac{U_i}{(P_x+P_z)} = \frac{1}{4} , \tag{29b}$$

and

$$\frac{U_e}{(P_x+P_z)} = \frac{1}{4} . \tag{29c}$$

This known power partitioning [9] is characterized by a large magnetic field energy dissipation by both electrons and ions.

B. The 2D limit

After examining the 1D limit, at which the shocks are reduced to the known 1D shocks [11], we turn to the 2D limit, and recover the recently discovered [10] 2D shocks. At the limit opposite to the 1D limit,

$$u \gg u_s . \tag{30}$$

At this limit, 2D effects play the major role. Then

$$P_x = -\left[\frac{b_u^2(1+b_u^3)}{6} - \frac{(1+b_u^5)}{10} \right] \frac{8u}{(1-b_u)^2(1+b_u)} \tag{31a}$$

and

$$P_z \ll P_x . \tag{31b}$$

The main flux of magnetic field energy is radial, perpendicular to z, the direction of shock propagation. This feature is common to the wave penetration due to the Hall field [8]. We again examine the two cases, a weak shock and a strong shock. If the shock is weak and Eq. (24) holds,

$$1+b_u \ll 1 ,$$

the radial flux of magnetic field energy, expressed in Eq. (31a), becomes

$$P_x = u(1+b_u) , \tag{32}$$

while P_z satisfies Eq. (31b). The flux of magnetic field energy is larger than it is at the 1D limit. The partitioning of the power is also described here by Eq. (27). The dissipated magnetic field energy is small. If the upstream plasma is unmagnetized,

$$b_u = 0 ,$$

the radial flux of the magnetic field energy, expressed in Eq. (31a), becomes

$$P_x = \tfrac{4}{5} u . \tag{33}$$

The partitioning of the power is then

$$\frac{U_B}{(P_x+P_z)} = \tfrac{5}{8} \tag{34a}$$

and

$$\frac{U_e}{(P_x+P_z)} = \tfrac{3}{8} , \tag{34b}$$

while

$$U_i \ll U_e, U_B . \tag{34c}$$

The work done on the ions is small. The flux of magnetic field energy is distributed between the magnetic field energy built in the shock downstream and the energy dissipated as work done on the electrons.

IV. THE VELOCITY AND THE STRUCTURE OF THE SHOCK

An implicit expression for the shock velocity is obtained by integration of Eq. (22):

$$\tfrac{1}{2} \ln \left[\frac{(u-u_s)(u_0+u_s)}{(u+u_s)(u_0-u_s)} \right] + \frac{u_s}{u} - \frac{u_s}{u_0} = -\frac{x u_s}{2(1-u_s^2)} , \tag{35}$$

where u_0 is the shock velocity at $x=0$. The shock velocity has to satisfy the relation

$$u \ge u_s(1+b_u)^{1/2} , \tag{36}$$

otherwise the expression for the density, Eq. (9), becomes negative. When 2D effects are small,

$$u_0 = u_s(1+\delta), \quad \delta \ll 1 , \tag{37}$$

the shock velocity is

$$u = u_s \left\{ 1+\delta \exp \left[-\frac{x u_s}{(1-u_s^2)} \right] \right\} . \tag{38}$$

When the 2D effects are pronounced,

$$u_0 \gg u_s , \tag{39}$$

the shock velocity is

$$u^3 = u_0^3 \Bigg/ \left[1+\frac{3}{2}\frac{x u_0^3}{(1-u_s^2)u_s^2} \right] . \tag{40}$$

These are the recently discovered high velocity shocks [10].

For the analysis of the 2D flow we specify, in addition to the upstream parameters (b_u or u_s) and the single downstream parameter ($b=-1$), the velocity of the shock u_0 at $x=0$. The shock velocity $u(x)$ and the partitioning of the power are then determined, with the exception of the partitioning of the electron kinetic energy between thermal energy and directed energy.

We turn to the shock structure. As is often so [6,10,11,15,16], when the collisionality is small, the electron inertia determines the shock structure. Seeking an evolution in which the magnetic field depends on

$$\xi \equiv [z-u(x)\tau] f(x) , \tag{41}$$

we write Eq. (18) as

$$\frac{\epsilon f^2}{n}\frac{d}{d\xi}\left[\frac{1}{n}\frac{db}{d\xi}\right]+\frac{\epsilon}{2u}\left[n f^2(b^2-b_u^2)\frac{d}{dx}\left[\frac{1}{u^2}\right]-\frac{d}{dx}(f^2)\right]\frac{1}{n^2}\left[\frac{db}{d\xi}\right]^2-\frac{\nu f}{un}\frac{db}{d\xi}+F(b)=0 . \tag{42}$$

Furthermore, by defining the Lagrangian coordinate

$$d\zeta(x) \equiv d\xi\, n(\xi,x)/(\sqrt{\epsilon} f(x)) , \tag{43}$$

and the normalized resistivity

$$\eta \equiv \nu/(\sqrt{\epsilon} u) , \tag{44}$$

where the coordinate ζ is measured in units of the local electron skin depth, we transform Eq. (42) to

$$\frac{d^2b}{d\zeta^2}+g(b)\left[\frac{db}{d\zeta}\right]^2-\eta\frac{db}{d\zeta}+F(b)=0 . \tag{45}$$

Here

$$g(b) \equiv -2\frac{d}{dx}\left[\frac{1}{u}\right]\frac{(b^2-b_u^2)}{(b^2-b_u^2-2u^2)}-\frac{1}{2u}\frac{d}{dx}(\ln f) . \tag{46}$$

In addition to b_u and u, we still have the freedom to choose $d/dx(\ln f)$. All these parameters determine the shock structure and the partitioning of the electron energy.

Similarly to the analysis of Sagdeev [11], it is possible to introduce an effective potential that determines the particle "coordinate" as a function of "time." This can be done by defining

$$q(b) \equiv \exp\left[-\int_{b_u}^{-1} db\, g(b)\right] \times \int_{b_u}^{b} db' \exp\left[\int_{b_u}^{b'} db'' g(b'')\right] . \tag{47}$$

Equation (44) is transformed to

$$\frac{d^2q}{d\zeta^2}-\eta\frac{dq}{d\zeta}+\mathcal{F}=0 , \tag{48}$$

where

$$\mathcal{F} \equiv \exp\left[-\int_{b}^{-1} db' g(b')\right] F . \tag{49}$$

Equation (47) describes in a standard manner the motion with a friction of a particle in a potential well, where the potential is

$$P=\int_1^q dq' \mathcal{F} . \tag{50}$$

From the analog to the particle in the potential well, it follows that

$$P[q(b=-1)]=\int_{-\infty}^{\infty} d\zeta\, \eta \left[\frac{dq}{d\zeta}\right]^2 . \tag{51}$$

For investigating the partitioning of power we find Eqs. (47)–(51) less useful. We pursue the analysis of Eqs. (45) and (46).

V. THE ELECTRON ENERGY

We would like to calculate Q, the rate of Joule heating. Using Eqs.. (18b), (41), and (43), we write

$$Q=\int_{-\infty}^{\infty} d\zeta\, \eta \left[\frac{db}{d\zeta}\right]^2 . \tag{52}$$

We examine the penetration into an unmagnetized plasma $b_u=0$, at which the work on the electrons is large. Also, we assume that 2D effects are important:

$$u \gg 1 . \tag{53}$$

Following these assumptions, we approximate $g(b)$ as

$$g(b) \cong g \equiv -\frac{1}{2u}\frac{d}{dx}(\ln f) , \tag{54}$$

which is independent of b. Let us assume that

$$|g| \gg 1 \ . \quad (55)$$

In Eq. (45) there are now two parameters: g and η. we neglect the first term on the RHS of that equation and obtain

$$p\left[\frac{db}{ds}\right]^2 - \frac{db}{ds} + F(b) = 0 \ , \quad (56)$$

where

$$s \equiv \zeta/\eta, \quad p \equiv g/\eta^2 \ . \quad (57)$$

The equation in this form is characterized by the parameter p only. In the regime in which $-1 \le b \le 0$, the value of F is positive. If g is negative, p is negative as well. We solve Eq. (56) and find that

$$\frac{db}{ds} = \frac{1-(1-4pF)^{1/2}}{2p} \ . \quad (58)$$

Since we have an expression for the derivative of b as a function of s, we change the variable of integration in Eq. (52) from ζ to b and express the rate of heating as

$$Q = \int_{-1}^{0} db\, \eta \frac{db}{d\zeta} \ . \quad (59)$$

Using Eqs. (57) and (58), we obtain that

$$Q = \int_{-1}^{0} db \left[\frac{1-(1-4pF)^{1/2}}{2p}\right] \ . \quad (60)$$

A. Large electron heating

We now examine the asymptotic limits. We first look at the case

$$|p| \ll 1 \ . \quad (61)$$

We then approximate

$$\frac{db}{ds} \cong F \ , \quad (62)$$

and, therefore, the heating is

$$Q = \int_{-1}^{0} db\, F(b) \ , \quad (63)$$

Using the inequality (53), we approximate the potential as

$$F(b) = -b - b^4 \ , \quad (64)$$

where $b_u = 0$ and we neglect the second term in Eq. (20). We obtain

$$Q = 0.3 \quad (65)$$

in agreement with Eqs. (33) and (34b). Inequality (61), therefore, defines the collisional regime, in which the Joule heating is large and

$$U_e \cong uQ \gg A \ . \quad (66)$$

B. Small electron heating

We turn to the collisionless case

$$|p| \gg 1 \ . \quad (67)$$

Here we approximate

$$\frac{db}{ds} \cong \left[\frac{-F}{p}\right]^{1/2} \ . \quad (68)$$

The heating is

$$Q \cong \left[-\frac{1}{p}\right]^{1/2} \int_{-1}^{0} db\, F^{1/2} = 0.524 \left[-\frac{1}{p}\right]^{1/2} \ . \quad (69)$$

The integral in Eq. (69) was calculated numerically.

C. The limit of zero resistivity

The approximation (56) fails when the velocity $db/d\xi$ becomes small, in which case the second term in Eq. (45) becomes smaller than the first term:

$$\left| g \left[\frac{db}{d\zeta}\right]^2 \right| \ll \left| \frac{d^2 b}{d\zeta^2} \right|, |F(b)| \ . \quad (70)$$

Let us estimate the contribution to the dissipation of these small oscillations. When both inequality (70) holds and the resistivity is small,

$$\frac{1}{2}\left[\frac{db}{d\zeta}\right]^2 \cong \int_{-1}^{b_A} db\, F(b) \ , \quad (71)$$

where b_A is the value of the magnetic field at which the derivative of b vanishes. To be consistent with the assumption (70), we require that

$$2g \int_{-1}^{b_A} db\, F(b) \ll F(b_A) \ . \quad (72)$$

We linearize $F(b)$ near $b = -1$, as

$$F(b) \cong 3(1+b) \ , \quad (73)$$

and, as a result, inequality (72) becomes

$$|1 + b_A| \ll \frac{1}{|g|} \ . \quad (74)$$

Thus, when Eqs. (70)–(74) hold, the heating is

$$Q \cong \int_{-1}^{b_A} db\, F(b) \ , \quad (75)$$

which, for $b_A \cong -1$, and with Eq. (74), is

$$Q \cong 1.5(1+b_A)^2 = \frac{1.5}{g^2} \ . \quad (76)$$

When Q calculated by Eq. (76) is larger than Q calculated by Eq. (69), the heating is expressed by Eq. (76). This happens when the resistivity is so small that

$$\eta \ll \frac{2.86}{|g|^{3/2}} \ . \quad (77)$$

D. The heating in the three regimes

As we described in Sec. V A, when the resistivity is so large that Eq. (61) is valid, or, specifically, if Eq. (57) is used, when

$$\eta \gg |g|^{1/2} , \quad (78)$$

the heating is given by Eq. (65):

$$Q = 0.3 .$$

From Eqs. (57), (67), (69), and (77), we conclude that for a small resistivity

$$\frac{2.86}{|g|^{3/2}} \ll \eta \ll |g|^{1/2} \quad (79)$$

the heating is

$$Q = 0.524 \frac{\eta}{|g|^{1/2}} . \quad (80)$$

At the limit of zero resistivity, when Eq. (77) is valid,

$$\eta \ll \frac{2.86}{|g|^{3/2}} ,$$

the heating is described by Eq. (76):

$$Q \cong \frac{1.5}{g^2} .$$

Equation (76) expresses the heating at the collisionless limit, i.e., $Q(g, \eta \to 0)$.

Let us apply this analysis to the self-similar evolution demonstrated in Ref. [10]. In the self-similar magnetic field evolution [10],

$$u \cong x^{-1/3}, \quad f = x^{1/3} , \quad (81)$$

and

$$g \cong x^{-2/3} \gg 1 . \quad (82)$$

Equation (78) becomes

$$\frac{\nu_e}{\omega_H} \gg x^{-2/3} , \quad (83)$$

where ν_e and ω_H are the dimensional electron collision frequency and electron-ion hybrid cyclotron frequency. Equations (79) and (80), respectively, become

$$2.86 x^{2/3} \ll \frac{\nu_e}{\omega_H} \ll x^{-2/3} \quad (84)$$

and

$$Q = 0.524 \frac{\nu_e}{\omega_i} . \quad (85)$$

Here ω_i is the ion cyclotron frequency. Equations (77) and (76), respectively, are

$$2.86 x^{2/3} \gg \frac{\nu_e}{\omega_H} \quad (86)$$

and

$$Q \cong \frac{1.5}{x^{4/3}} . \quad (87)$$

VI. NUMERICAL EXAMPLES AND CONCLUSIONS

In this section we give numerical examples of the shock structure and the distribution of the electron energy between heating and convection. In the examples the shock velocity is high, corresponding to 2D shocks. The governing equation is Eq. (45), where $g(b)$ is approximated as a constant, according to Eq. (54), and where $F(b)$ is approximated by Eq. (64). Therefore, the flux of mag-

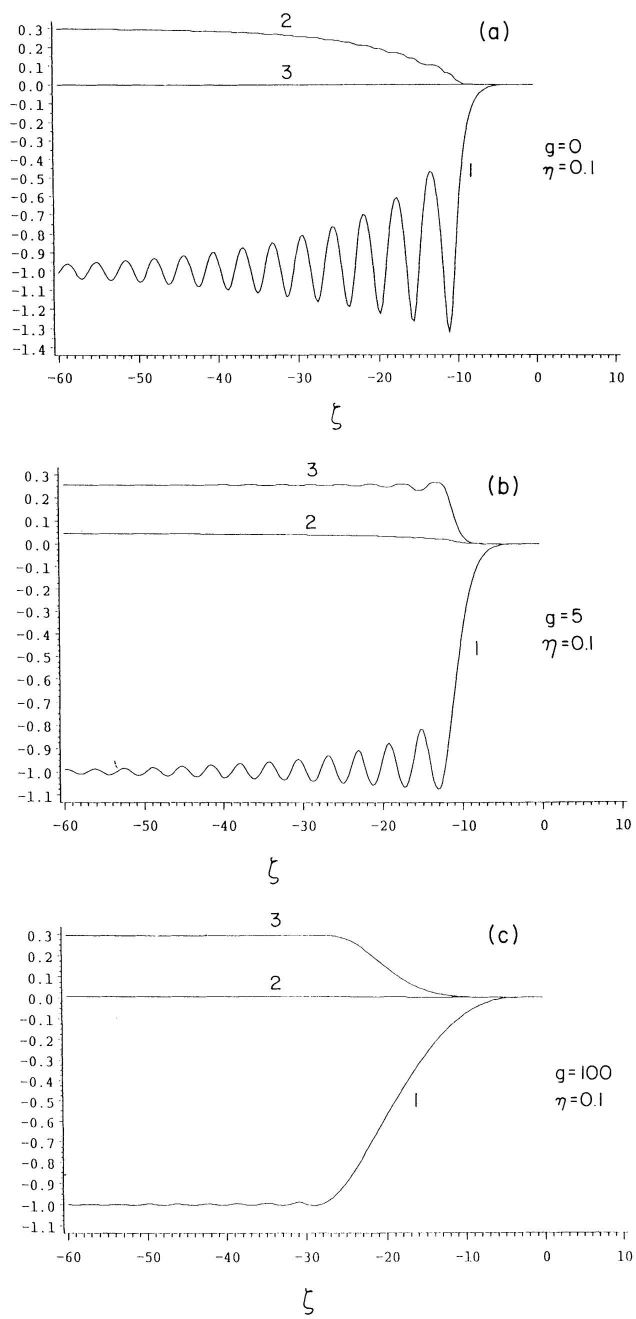

FIG. 1. The shock structure and the electron heating for (a) $g = 0$, (b) $g = 5$, and (c) $g = 100$. Curve 1 shows b, curve 2 shows q [defined in Eq. (88)], and curve 3 shows C [defined in Eq. (89)]. The calculation was done by solving Eq. (45) with the approximations in Eqs. (54) and (64). Here $\eta = 0.1$.

netic field energy is distributed between the magnetic field energy in the shock downstream [Eq. (34a)] and work on the electrons [Eq. (34b)], while the work on the ions is small [Eq. (34c)]. In the examples we examine the influence of the parameters g and η.

Figures 1(a)–1(c) show the magnetic field b, the Joule heating, and the quantity C as a function ζ. The heating is given by the integral

$$q(\zeta)=\int_{\zeta}^{\infty} d\zeta' \eta \left[\frac{db}{d\zeta'}\right]^2 . \tag{88}$$

The quantity C is

$$C(\zeta)=\int_{-\zeta}^{\infty} g d\zeta' \left[\frac{\partial b}{\partial \zeta'}\right]^3 . \tag{89}$$

At the limit $\zeta \rightarrow \infty$ the quantity C equals A/u, where A is defined in Eq. (17a). In Figs. 1(a)–1(c) the resistivity was taken to be $\eta=0.1$. In Fig. 1(a), $g=0$, in 1(b) $g=5$, and in 1(c) $g=100$. In all three figures the shock structure includes damped spatial oscillations. The amplitude of the oscillations is smaller for a larger g, and in Fig. 1(c), where g is 100, the amplitude of the oscillations is very small. In Fig. 1(a) all the work done on the electrons goes to electron heating. No energy is convected away. In Fig. 1(b) most of the energy is convected away and only a small fraction heats the electrons. In Fig. 1(c) the electron heating is too small to be noticed.

The analysis of the electron heating presented in Sec. V is demonstrated in Figs. 2–4. The heating Q expressed in Eq. (52) was calculated through a numerical solution of Eq. (45) with the approximation (64) for $F(b)$. Figure 2 shows Q as a function of g and η. It is seen in the figure that for $g=0$ the heating is 0.3, irrespective of the value of η. For $g \neq 0$ the heating for $\eta=0$ is smaller than 0.3, but becomes 0.3 for large η. The value of Q decreases with increasing g.

The detailed dependence of the heating Q on the resistivity η is shown in Fig. 3. Plotted is $\ln(Q)$ as a function of $\ln(\eta)$ for $g=25$. The straight line shows $\ln[0.524\eta/(-g)^{1/2}]$ as a function of $\ln(\eta)$. It is seen in the figure that for large and small values of η the heating

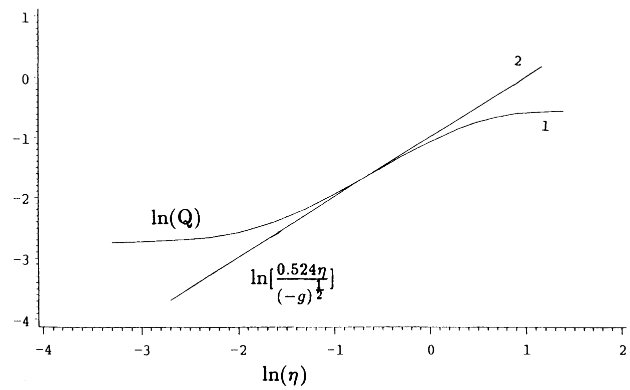

FIG. 3. The rate of heating Q [Eq. (52)] as a function of the normalized resistivity η on a natural logarithmic scale (curve 1). The calculation was done by solving Eq. (45) with the approximations in Eqs. (54) and (64). Curve 2 shows the approximate rate of heating as given by Eq. (80). Here $g=25$.

is constant. In fact it is easy to verify that it is approximately described by Eqs. (65) when η is large, and by Eq. (76) when η is small. In the intermediate regime the two lines coincide and Eq. (69) holds.

Figure 4 shows the heating as a function of g at the limit of zero resistivity. For each value of g the value of Q was calculated for smaller and smaller values of η, until convergence was reached. Plotted is the product of Q by g^2, which approaches a constant for a large g. This constant is approximately 1.2, somewhat smaller than 1.5, the number we estimated in Eq. (76).

In conclusion, we have described a different class of 2D shocks in plasmas. At the 1D limit shocks in cold plasmas are recovered. Generally, the 2D shocks have a velocity that is higher than the velocity of the 1D shocks. At the limit opposite the 1D limit, the shock velocity is high and the flux of magnetic field energy is distributed between building the magnetic field in the shock downstream and work on the electrons. Only a small part is converted into work on the ions.

A subclass of the class of fast shocks includes shocks in which the electron heating is small. We have presented

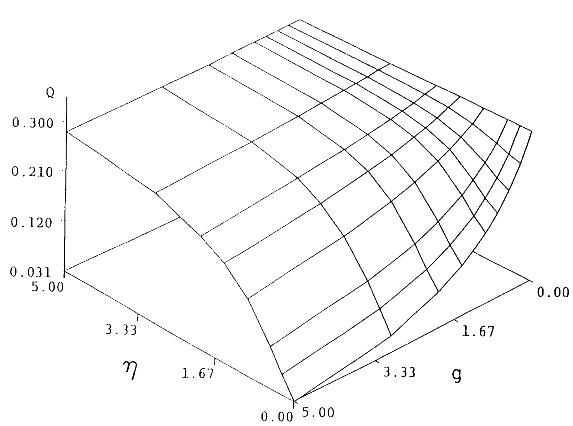

FIG. 2. The rate of heating Q [Eq. (52)] as a function of g and η. The calculation was done by solving Eq. (45) with the approximations in Eqs. (54) and (64).

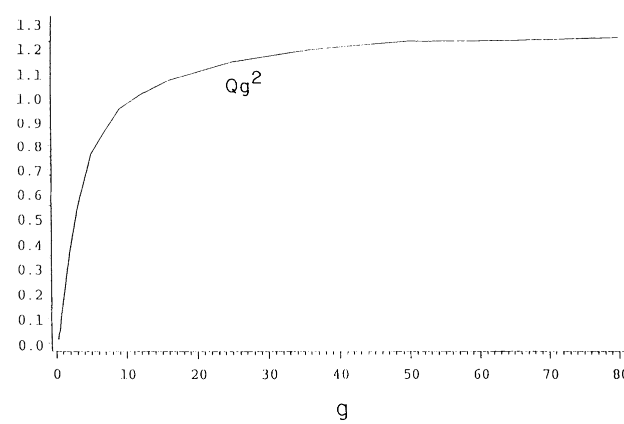

FIG. 4. The product Qg^2 as a function of g at the limit $\eta \rightarrow 0$. The calculation of Q was done by solving Eq. (45) with the approximations in Eqs. (54) and (64).

here a detailed analysis of the distribution of energy supplied to the electrons, and have demonstrated the possibility of shocks in which the dissipated energy is convected and does not heat the electrons in the shock downstream.

ACKNOWLEDGMENTS

One of us (L.I.R.) thanks the Einstein Center for Theoretical Physics for supporting his visit to the Weizmann Institute.

[1] C. K. Chu and R. A. Gross, in *Advances in Plasma Physics,* edited by A. Simon and W. B. Thompson (Interscience, New York, 1969), Vol. 2, p. 139.

[2] A. S. Kingsep, Yu. V. Mokhov, and K. V. Chukbar, Fiz. Plazmy **10**, 584 (1984) [Sov. J. Plasma Phys. **10**, 495 (1984)].

[3] K. V. Chukbar and V. V. Yankov, Zh. Tekh. Fiz. **58**, 2130 (1988) [Sov. Phys. Tech. Phys. **33**, 1293 (1988)].

[4] A. V. Gordeev, A. V. Grechikha, A. V. Gulin, and O. M. Drozdova, Fiz. Plazmy **17**, 650 (1991) [Sov. J. Plasma Phys. **17**, 381 (1991)].

[5] A. Fruchtman, Phys. Fluids B **3**, 1908 (1991).

[6] L. I. Rudakov, C. E. Seyler, and R. N. Sudan, Comments Plasma Phys. Controlled Fusion **14**, 171 (1991).

[7] B. V. Oliver, L. I. Rudakov, R. J. Mason, and P. L. Auer, Phys. Fluids B **4**, 294 (1992).

[8] A. Fruchtman and K. Gomberoff, Phys. Fluids B **4**, 117 (1992).

[9] A. Fruchtman, Phys. Fluids B **4**, 855 (1992).

[10] A. Fruchtman and L. I. Rudakov, Phys. Rev. Lett. **69**, 2070 (1992).

[11] R. Z. Sagdeev, in *Reviews of Plasma Physics,* edited by M. A. Leontovich (Consultants Bureau, New York, 1966), Vol. 4, p. 23.

[12] C. W. Mendel, Jr. and S. A. Goldstein, J. Appl. Phys. **48**, 1004 (1977).

[13] P. F. Ottinger, S. A. Goldstein, and R. A. Meger, J. Appl. Phys. **56**, 774 (1984).

[14] M. Sarfaty *et al.* (unpublished).

[15] Ya. L. Kalda and A. S. Kingsep, Fiz. Plazmy **15**, 874 (1989) [Sov. J. Plasma Phys. **15**, 508 (1989)].

[16] L. I. Rudakov, Plasma Phys. Rep. **19**, 433 (1993).